problem. Errors in pressure prediction will affect drag predictions much more than lift predictions because leading edge suction is an important factor in resultant pressure drag.

In conclusion, it is shown that good estimates of pressure distributions can be derived from the combination of calculation procedures presented here. Also accurate lift and drag predictions can be made up to moderate angles of attack in transonic mixed flow.

References

[1] Sinnott, C. S. and Osborne, J., "Review and Extension of Transonic Aerofoil Theory," R & M 3136, 1961, British Aeronautical Research Council.

[2] Thompson, N. and Wilby, P. G., "Leading Edge Supersonic Velocity Peaks and the Determination of the Velocity Distribution on an Aerofoil in a Sonic Stream," Proceedings of AGARD Conference on Transonic Aerodynamics, AGARD C.P. 35, 1968.

[3] Theodorsen, T., "Theory of Wing Sections of Arbitrary Shape," Rept. 411, 1931, NACA.

[4] Wilby, P. G., "The Calculation of Subcritical Pressure Distributions on Symmetric Aerofoils at Zero Incidence," Rept. 1208, 1967, British National Physical Laboratory.

[5] Pearcey, H. H., "The Aerodynamic Design of Section Shapes for Swept Wings," *Advances in Aeronautical Sciences*, Vol. 3, 1960, p. 292.

[6] Pankhurst, R. C. and Squire, H. B., "Calculated Pressure Distributions for the RAE 100-104 Aerofoil Sections," TN Aero 2039, 1950, British Royal Aircraft Establishment.

[7] Abbott, I. H. and Von Doenhoff, H. E., *Theory of Wing Sections*, Dover, New York, 1959, p. 313.

[8] Antonio, F., "Completed Tabulation in the U.S. of Tests of 24 Airfoils at High Mach Numbers," ACR No. L53E21, 1945, NACA.

Severity Comparisons of Specified and Actual Impulse Tests

KENNETH A. STEAD*

University of Mississippi, University, Miss.

Introduction

MANY systems and components are required to withstand a certain impulse test, the application of several to many short term, relatively large, forces. Because of the equipment limitations, the specified impulses sometimes cannot be duplicated, and the question arises: Is the component satisfactory or not? Solutions exist for some configurations, but they tend to be sophisticated and the testing engineer may not have time to familiarize himself with them. Even more important, there are a multitude of configurations which defy analysis, for which no solutions are available. Included in the latter are bellows, valves, pipe lines with bends, etc. The exposition below develops a simple method of determining, in some cases, an absolute answer to the question, and in any case, an aid to engineering judgement.

The assumptions and limitations of the analysis are 1) maximum displacement is the basic measure of severity, and this is not always the case, but stress wave and stability solutions are rare and offer no hope of being generalized; 2) it is supposed that small, linear vibrations ensue; 3) the only differences between the specified and actual impulses are in their magnitudes and time profiles, and the test force or pressure is applied in the same place and with the same relative distribution as that specified; 4) the first few resonant frequencies have been found by experiment, applying the force in the same place as during the impulse test; 5) the specified and test impulses are periodic.

Received December 10, 1969.

* Assistant Professor, Department of Civil Engineering.

Development

For small vibrations of an elastic system, there exists[1] a set of normal generalized coordinates such that Lagrange's equations of motion reduce—in the absence of friction—to a set of uncoupled differential equations

$$q_i + P_i^2 q_i = Q_i(t)/m_i \tag{1}$$

in which q_i is the ith generalized coordinate, P_i is the natural frequency associated with the ith natural mode of vibration, Q_i is the generalized force corresponding to the ith mode, and m_i is generalized mass. For a continuous body or system there are an infinity of coordinates and, hence, an infinity of Eq. (1). As is well known, for practical purposes, only a few of the lower modes need be considered.

The forcing functions, both specified and test, are supposed to be arbitrary except for periodicity. Hence, Q_i is conveniently expressed as

$$Q_i(t) = Q_{i0}\left[a_0 + \sum_{n=1}^{\infty} (a_n \cos nz + b_n \sin nz)\right] \tag{2}$$

in which Q_{i0} is the maximum value of the generalized force Q_i; a_0, a_n, and b_n are the Fourier coefficients for a function $f(z)$ having the same time profile as the forcing function with maximum amplitude unity, and $z = \omega t = 2\pi t/T$ where ω is the frequency and T is the period of the forcing function.

Since we are dealing with many pulses, and the natural vibrations induced quickly become insignificant even with small damping, we need only the particular solution of Eq. (1). Substituting Eq. (2) into Eq. (1) and defining

$$\beta_i = \omega/P_i \tag{3}$$

the particular solution of Eq. (1) is

$$q_i = \frac{Q_{i0}}{m_i P_i^2}\left[a_0 + \sum_{n=1}^{\infty} \frac{1}{(1 - n^2\beta_i^2)} (a_n \cos nz + b_n \sin nz)\right] \tag{4}$$

The absolute maximum of (4) is desired. For most specified forcing functions, it will be clear that this maximum is attained when the series sum is either

$$[\text{sum}] = a_0 \pm \sum_{n=1}^{\infty} \frac{a_n}{(1 - n^2\beta_i^2)} \tag{5a}$$

or

$$[\text{sum}] = \pm \sum_{n=1}^{\infty} \frac{b_n}{(1 - n^2\beta_i^2)} \tag{5b}$$

This is not the case for the forcing function actually applied in the test. It is likely neither even nor odd, and the angles nz for which the series sum is a maximum not obvious. One might take d/dz of Eq. (4), equate it to zero, and solve for the angles at which the relative maxima and minima occur. Generally this will be a trial and error process requiring a series sum for each trial. It is probably less time consuming and certainly simpler to calculate the series sum for close-spaced arguments, say $\pi/18$, and take the absolute maximum found as the required sum.

Supposing the maximum absolute value of the series to be found, the maximum value of the ith coordinate, q_i is written

$$\bar{q}_i = (Q_{i0}/M_i P_i^2)\ [\text{sum}] \tag{6}$$

Denoting quantities associated with the test with superscript t and those associated with the specifications with superscript s, the ratio of the maximum value of the ith coordinate produced by the test to that which would have been produced by the specification impulses is

$$\bar{q}_i{}^t/\bar{q}_i{}^s = Q_{i0}{}^t[\text{sum}]^t/Q_{i0}{}^s[\text{sum}]^s \quad (7)$$

Although the form of the generalized forces is unknown, since it is supposed that there are no spatial differences between the specified and test forcing functions, the ratio $Q_{i0}{}^t/Q_{i0}{}^s$ equals the ratio of the maximum force or pressure of the test to that specified. These latter quantities are known. Denoting them by F^t and F^s, we have

$$\bar{q}_i{}^t/q_i{}^s = F^t[\text{sum}]^t/F^s[\text{sum}]^s \quad (8)$$

If Eq. (8) ≥ 1 for the ith mode, the test impulses generated more excursion in that mode than the specified impulses would have. If Eq. (8) ≥ 1 for all i, the test is clearly as severe as that specified. If Eq. (8) < 1 for all i, the test is clearly not as severe as that specified.

Discussion

For those cases when Eq. (8) ≥ 1 for some modes and <1 for others, judgement must still be applied. However, it is felt that knowing these ratios for the first few modes is a great advantage over guesswork. Perhaps a reasonable criterion would be that the average of Eq. (8) over the known modes be at least one.

Reference

[1] Strutt, J. W., "3rd Baron Rayleigh," *The Theory of Sound*, Vol. 1, Art. 87, Dover, New York, 1945.

Sonic Boom Minimization Schemes

David Siegelman*
Avco Systems Division, Wilmington, Mass.

RECENTLY, it has been suggested that desirable modifications of objectionable sonic boom pressure signatures may be accomplished by the addition of mass or energy or by electro-aerodynamic means. The possibility of successfully utilizing any of these schemes is a matter of much current controversy.[1–3] It is the purpose of this Note to present simple analytical techniques with which proposals involving mass or energy addition may be evaluated and to infer some preliminary results concerning their feasibility.

The objective of the proposed mass or energy addition schemes is to create a "phantom" boundary which will favorably alter the effective area distribution of a given airplane. Such favorable alterations may be designed to produce either plateau ($A_e \alpha X^{3/2}$) or finite rise time ($A_e \alpha X^{5/2}$) pressure signatures.[4] The effective area variation required of the addition scheme is, therefore, the difference in effective areas between the phantom and actual bodies

$$\Delta A(X) = A_{\text{PHANTOM}}(X) - A_{\text{ACTUAL}}(X) \quad (1)$$

Assuming for the time being that this distribution has been selected, the problem becomes one of relating the required area growth to a causal mass or energy distribution. Identifying the phantom boundary as the "dividing streamline" for cases involving mass injection only, the mass distribution required to produce a given variation in area under the flight conditions of interest here can be determined by application of the results of slender body theory[5]

$$\dot{m}(x) = \rho_\infty U_\infty \int_0^x (1 + \tfrac{1}{2}M_\infty{}^2 Cp\delta t)\Delta A'(t)dt \quad (2)$$

where the pressure coefficient is explicitly related to the area distribution by Eq. (9.34C) of Ref. 5.

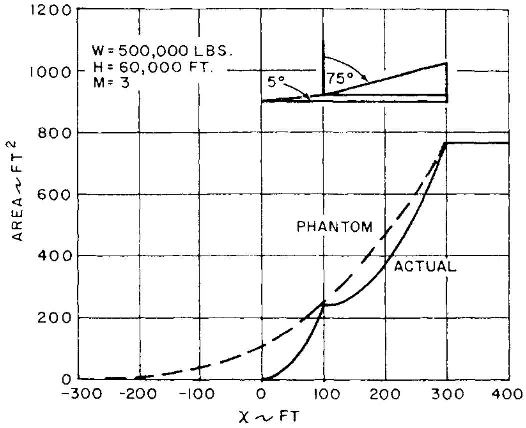

Fig. 1 Effective area distributions of actual configuration and phantom boundary.

The analysis of energy addition schemes is more complex and the model adopted should, strictly speaking, be dependent upon the proposed manner in which the energy is to be added (conduction, convection, radiation). The problem may be viewed as essentially an inviscid interaction problem in which the flow within a reference streamtube tries to expand in area (due to heat addition) against a self-induced, retarding pressure gradient. (It is assumed that thermal layer growth ($\alpha X^{1/2}$ or $X^{0.8}$) will not be sufficiently rapid to be important in this application).

The inner (duct-like) flow can be described by the one-dimensional flow equations

$$(d/dx)(\rho u A) = 0 \quad (3a)$$

$$\rho u(du/dx) + (dP/dx) = 0 \quad (3b)$$

$$\frac{\gamma}{\gamma - 1}\rho u_A\left(\frac{1}{\rho}\frac{dP}{dx} - \frac{P}{\rho^2}\frac{d\rho}{dx} + \frac{\gamma - 1}{\gamma}u\frac{du}{dx}\right) = \dot{q}(x) \quad (3c)$$

Equations (3) are 3 equations involving 5 quantities. The additional relations required for closure of the system may be obtained from 1) the outer flow, where it is required that the axial pressure distribution and area variation of the reference streamtube be related[5] and 2) the requirement that the area variation be that given by Eq. (1). Cases involving both heat and mass addition would require the addition of a source term in the continuity equation plus one more relation (such as specifying either $\dot{m}$ or $\dot{q}$ and solving for the other or giving a functional relation between $\dot{m}$ and $\dot{q}$).

With these analytical models available, the mass or energy requirements to suitably modify the effective area distribution of a simple cone-cylinder-subsonic leading edge delta wing configuration can be estimated. The effective area of this configuration can be obtained analytically[6] and is shown in Fig. 1. (Wing-body interference effects were neglected and only normal "cutting planes" were considered.) A finite-rise-time bow shock modification requirement was postulated and the resulting minimum length phantom body effective area curve, also shown in Fig. 1, was found. This minimum length phantom was about 255 ft longer than the reference aircraft; shorter phantoms being prohibited since they resulted in negative areas for the axisymmetric "blown" body. The blown body area distribution, from Eq. (1), is given in Fig. 2. The mass addition analysis can be applied in an approximate manner (by neglecting the variation of density) and results in an approximate mass flow distribution given by

Received November 5, 1969.
* Staff Scientist. Member AIAA.